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NORMAL HEAT SOURCES AND SINKS IN THE LOWER TROPOSPHERE IN WINTER

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ABSTRACT

A synthesis is made of several published studies of the normal heat budget of the lower troposphere over the Northern Hemisphere in winter. Charts of heat sources and sinks are presented, based on two independent methods, one using the thermodynamic energy equation and the other a heat-balance procedure. Although there are differences in important details, both methods indicate that the horizontal scale of heating is the same as that of the normal lower-tropospheric temperature field, but that the field of heating is almost 90° out of phase with that of temperature.

This result has an important bearing on the energy budget of the circulation. It is concluded that the heat-balance method is correct in indicating that there is a positive correlation between the normal temperature and heating fields. According to the energy transformation equations, this means that there is a positive transformation from heating to potential energy at the scale of the climatological long waves. This energy is directly available for maintaining these waves against friction.

It is hoped that this study will be of some use in the design of numerical general circulation experiments, and as a basis of comparison with data from meteorological satellites.

1. INTRODUCTION

Attempts have been made by many investigators over the past 25 to 30 years to obtain an accurate picture of the distribution of sources and sinks of thermal energy which are responsible for driving the atmospheric circulation. Recent progress in improving the speed and capacity of electronic computers has raised the possibility of solving this problem, at least in principle, by including in a complex general-circulation model all the physical processes associated with heating and cooling. However, aside from the fact that these physical processes are poorly understood, there remains the difficulty that even the enormous speed and capacity of present machines are unequal to the task of resolving all of the complexities in a reasonable length of time. Therefore, in order to include some estimate of the heating fields in general circulation experiments, it has been necessary to employ simplified heating functions based largely on hypotheses arising from the judgment and experience of individual authors, as well as on the degree of complexity of their models (Phillips [25], Smagorinsky (to be published), Mintz [20], Charney [7], Fjørtoft [9]).

It is of vital importance that these estimates, though crude, give a reasonably accurate picture of the magnitude and pattern of the heat sources and sinks, because, as will be suggested at the conclusion of this article, their location relative to the different branches and space-time scales of the circulation is of critical importance in determining the sources of energy for, and therefore the life cycle of, these wind systems. Thus, there is evidently a need for a reasonably accurate picture of at least the normal fields of heating and cooling in the atmosphere so that these can be used as a check against the proposed heating functions.

There is also some evidence that the fields of heating and cooling are closely related to errors in current operational numerical circulation forecasts (Martin [19]). Therefore, a knowledge of how these fields are related to upper-level wave patterns should be of considerable practical assistance in routine forecasting.

Finally, it is desirable to provide a rational basis of comparison with the directly observed values of the global heat budget obtained from meteorological satellites.

For these three reasons the author was encouraged to attempt an up-to-date and, it is hoped, accurate synthesis of these older estimates of the normal heat sources and sinks in the lower troposphere over the Northern Hemisphere. Encouragement in this effort was provided through the recent publication of the results of two extensive studies on this subject (Staff Members, Academia Sinica [29], Budyko [5] [6]).

2. GENERAL PROCEDURE

The first step was to make a search of the literature to locate all published studies of heating and cooling covering as much of the Northern Hemisphere as possible. This task was rendered fairly simple because of the relatively small number of projects treating the subject on so vast a geographical scale. Despite this restriction, it is possible that some of the pertinent literature may have been overlooked. Furthermore, the reader is cautioned that the list of references at the end of this paper is not meant to be an exhaustive bibliography on the subject, but contains only the reports actually used in this project.

Next, the publications were divided into those in which the heating fields were determined using the thermodynamic energy equation, and those based on a heat-balance method. It was hoped in this way to obtain two independent estimates, thus providing an accuracy check. However, it was soon discovered that the final results of the two methods were not comparable, and, as will be discussed later, it was necessary to make a choice as to which gives the more reliable results.

After the pertinent reports for each of the two methods had been selected, the data were read from the published charts at each standard 10° intersection of latitude and longitude from the equator to 60° N., and converted to mean heating in the layer 1000 to 500 mb. expressed in c.g.s. units (cal. gm.$^{-1}$ sec.$^{-1}\times 10^{7}$). (It was found that not enough data were available north of 60° N. to obtain heating fields in the polar regions.) In only one case were original data tabulations used for extracting the needed information.

The effect of reading published charts, rather than copying data tabulations, is to decrease the magnitudes of the maxima and minima of heating.

It was found that there were available no more than two independent sources of information dealing with any one particular type of heating field. These were simply averaged together in constructing the final charts, with no attempt to weight them in accordance with their probable relative accuracy. To a certain extent such a screening for accuracy was taken care of in the original careful selection of source material.

In some cases, only one source of data was available; and in one case, none at all. Thus, some reworking and extension of the original data was inevitable in order to obtain complete hemispheric heating fields. It must be stated that this was done in some instances by the use of approximate techniques which would probably not have been deemed entirely valid by the original authors. These modifications will be discussed in some detail below.

The general forms of the equations used in the computations are presented in Sections 3 and 4. These will not be derived here, as this has been taken care of in some detail in the original source references. However, the major assumptions on which they are based, and the additional approximations used by the individual authors, will be treated here.

The final heating fields presented in figures 1 to 5 may be considered to apply to the colder season and are averages for the layer 1,000 to 500 mb. or 0 to 5.5 km. No distinction was made between individual data fields for winter (December, January, and February), and those for the month of January only. Examination of normal flow patterns shows that there are relatively small changes in the principal meteorological parameters (and presumably also in the heating field) between the months of December and February.

3. THERMODYNAMIC ENERGY EQUATION METHOD

From the first law of thermodynamics and the definition of potential temperature, the individual rate of heating per unit mass, averaged over a specified time interval, may be written as follows:

$$\overline{\frac{dq}{dt}}=c_p\frac{\partial \overline{T}}{\partial t}+c_p\overline{\mathbf{V}}\cdot\nabla\overline{T}+\frac{c_p\overline{T}}{\overline{\theta}}\overline{w}\frac{\partial\overline{\theta}}{\partial z}+c_p\overline{\mathbf{V}'\cdot\nabla T'}+\frac{c_p\overline{T}}{\overline{\theta}}\overline{w'\frac{\partial\theta'}{\partial z}}. \quad (1)$$

Here, q is the amount of heat per unit mass; c_p, specific heat at constant pressure; T, absolute temperature; t, time; $\mathbf{V}$, horizontal wind vector; ∇, horizontal gradient; θ, potential temperature; w, vertical wind component; and z, height. The bar overscore represents the arithmetic-averaged value of a given quantity for the selected time interval, and the prime symbol denotes the departure of a quantity from its time-averaged value.

The terms on the right of equation (1) may be designated, in order, the local heating (heat storage term), horizontal heating (horizontal advection of heat), vertical heating (vertical advection of heat), horizontal eddy-advection of heat, and vertical eddy-advection of heat.

The two principal assumptions made in deriving equation (1) are relatively minor. They are: (a) The local rate-of-change and horizontal advection of pressure are small compared to its vertical advection. (b) The quantity T/θ may be considered constant in time when used as a coefficient of the vertical heating term.

The first attempt to compute average heating from equation (1) was made by Wexler [31]. The results reported on here are based on the more recent studies of Aubert and Winston [2] and Staff Members, Academia Sinica [29], who computed the normal heating in the lower part of the troposphere for individual months or seasons of the year. Both of these studies omitted the two eddy terms and used a constant or normal vertical stability in evaluating $\partial\theta/\partial z$. They also used constant values of $\overline{\mathbf{V}}$, $\overline{T}$, and $\overline{w}$ within a finite vertical layer of the atmosphere. This is equivalent to assuming that there is no correlation in the vertical between the horizontal wind components and temperature, or between vertical motion and stability. Neither source made computations north of about 70° N. Few estimates were made by Aubert and Winston south of 20° N., and by Staff Members south of 10° N.

These two sources differ somewhat in other approximations used in their work as follows:

Aubert and Winston evaluated the heating for the layer sea level to 10,000 feet using normal sea level and 10,000-ft. charts. The gradient wind obtained from the average pressure field for this layer was used in computing $\overline{\mathbf{V}}$. The quantity $\overline{T}$ is the average mean virtual temperature between these two levels, and $\overline{w}$ was obtained from the divergence of the mean gradient winds. Their results must be questionable near high mountains, because pressures reduced to sea level were used, and no mountain-effect was included in the computation of $\overline{w}$.

The Staff Members used normal constant-pressure charts to evaluate heating in the layer 1,000 to 500 mb. (approximately sea level to 5.5 km.). The geostrophic wind at 700 mb. was used in computing $\overline{\mathbf{V}}$, and $\overline{T}$ was replaced by the thickness between the two layers. The mean vertical motion was obtained using the average divergence at the ground and at 500 mb. The authors state that the surface divergence was computed from observed winds, although it is difficult to see how this is possible in most areas of the globe, where no winds are available. The upper-level divergence was computed with the aid of the vertically integrated tendency equation. This seems to avoid the difficulty arising from the balance of terms in the divergence equation. The effect of topography was included in computing vertical motion at the ground, but this apparent improvement of the method of Aubert and Winston is largely nullified by the use of reduced 1000-mb. charts in computing thickness advection. Thus, their heating fields are probably also unreliable in mountain areas.

In order to obtain an average of these two heating fields, it was necessary to convert the values of mean heating in the layer 0 to 3 km., derived from data of Aubert and Winston, to the layer 0 to 5.5 km. This was done using the rather arbitrary assumption that the intensity of heating per unit mass varies linearly with height, reaching a maximum at 1 km. and vanishing at sea level and 10 km. Using this scheme, the ratio between average heating in the two layers can easily be obtained. Thus, the conversion was accomplished simply by dividing the values obtained from Aubert and Winston by 1.08.

The two independent estimates are remarkably similar (separate charts not shown). The major difference is in eastern Asia and the western Pacific, where the heating maximum computed by Staff Members is weaker and considerably farther to the east than that of Aubert and Winston. This discrepancy is difficult to trace, but may be due to the use of different normal circulation patterns.

The average of these two heating fields is shown in figure 1. A brief discussion of the pattern of heating and its relation to the broadscale temperature field will be deferred until Section 5. Here, an attempt will be made to assess its probable accuracy. This is not an easy matter, in view of the numerous assumptions discussed previously.

Inspection of figure 1 quickly reveals certain regions where one would suspect that the values are in error. For reasons given below, the indicated heating over eastern Canada and Siberia appears to be too large in comparison with that in the northwestern Atlantic and Pacific Oceans. Also, there is no apparent explanation for the small but positive heating over the oceans west of California and North Africa, which incidentally, appears on the charts of both authors. The northeastern continents and southeastern oceans in winter are known to be regions where precipitation is relatively light. This means that condensation, the only important source of heat in these areas, must be relatively weak. On the other hand, in both regions the air sometimes loses heat by contact with a colder land or ocean surface, and always loses heat by radiation to space.

Examination of the individual terms in equation (1) reveals the probable source of these errors. The reliability of the normal circulation patterns is probably quite acceptable, so that, except in the high mountain areas, the sum of the first two terms is reasonably correct. Thus the errors may be traced to the omission of the eddy terms and to the approximations used in computing vertical motion.

The necessity of accurate estimates of vertical motion stems from the fact that there is a strong negative correlation between the sum of the first two terms and the third term on the right side of equation (1). This has been demonstrated on a synoptic scale by Panofsky [24] who showed that vertical motions computed by the adiabatic method (essentially equation (1) with zero heating and the eddy terms omitted) are positively correlated with weather and with vertical motions computed from the divergence of observed winds (kinematic method). Several recent extensive studies of the kinetic energy

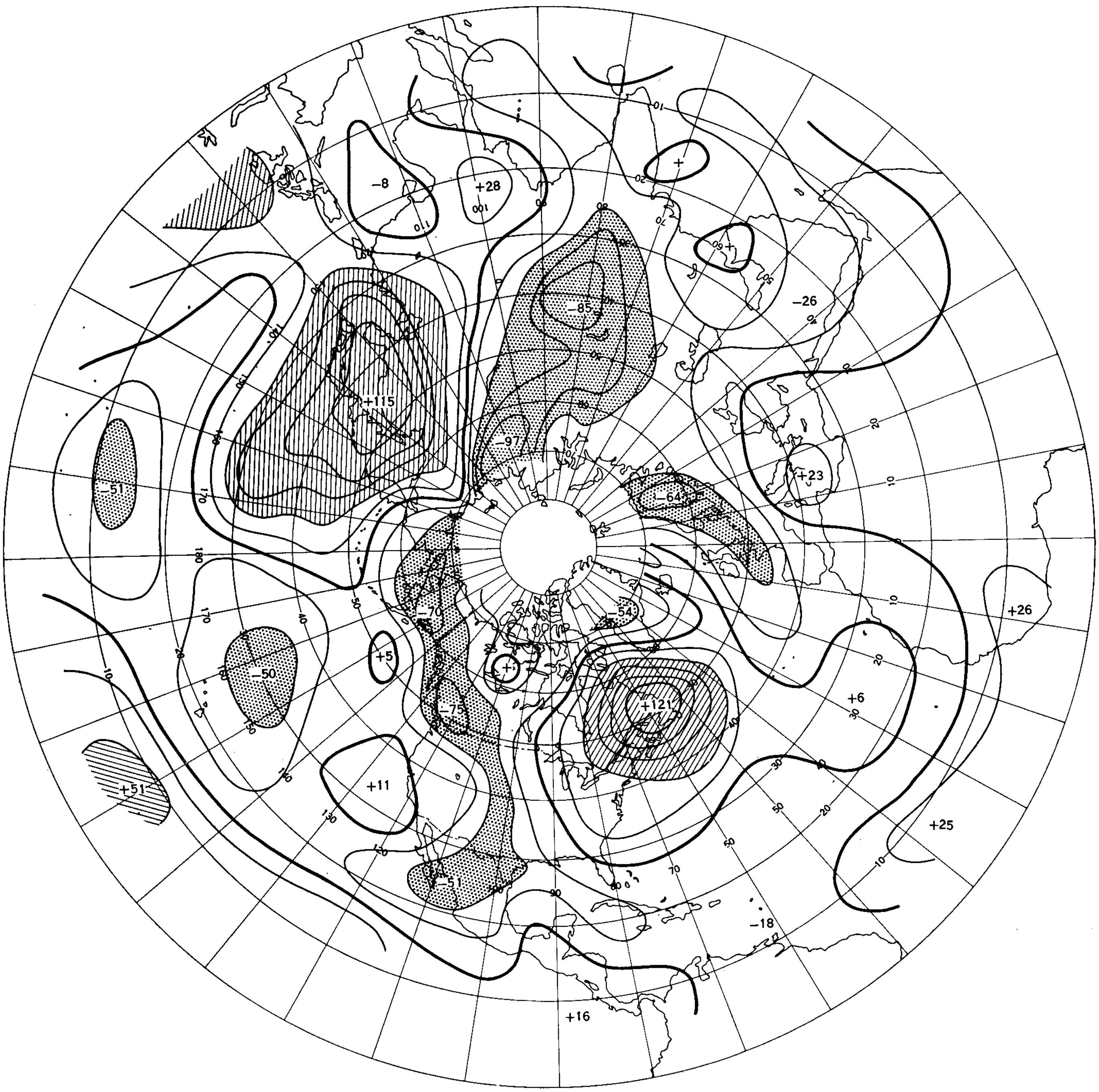

FIGURE 1.—Normal winter heating, sea level to 5.5 km., computed by the thermodynamic energy equation method. Units are cal. gm.$^{-1}$ sec.$^{-1}\times 10^{7}$. To convert to: cal. cm.$^{-2}$ day^{-1} multiply by 4; to C °. day^{-1}, by 4/100. Lines drawn for every 20 units with zero line heavier. Areas with heating greater than +40 units cross-hatched; cooling greater than −40 units stippled.

budget of the circulation have been based on vertical motions computed by the adiabatic method, evidently with some assurance that this gives reasonably accurate results, at least for that purpose (e.g., Jensen [15]).

Aubert and Winston [2] have shown that for nearly steady-state time-averaged circulations there is also a high negative correlation between the horizontal and vertical heating terms.

This tendency for a negative correlation between three of the important terms means that equation (1) must be used with extreme caution in estimating heating, since a large systematic error in any term may even lead to the wrong sign of the heating. This may result in serious errors in the spatial *pattern* as well as the *magnitude* of heating. This precaution applies especially to the term containing vertical motion, because all the known (indirect) methods of estimating the vertical wind component are subject to systematic errors.

In the two studies discussed here there is evidence that the computed vertical motions are too small. Indeed, Aubert and Winston show that for a few selected localities the mean monthly divergence (and therefore vertical motion) computed from observed winds is *four times* as large as the divergence of the gradient wind. This means that in general there is not enough compensation between the horizontal and the vertical terms. In regions of generally sinking motion in the northeastern continents and southeastern oceans this results in too much heating, while in the areas of slow mean upward motion in the western oceans, not enough heating is computed.

The omission of the eddy terms also has quite serious consequences in certain areas. Attempts to estimate the horizontal eddy effect (fourth term on the right of equation (1)) by an approximate theory of horizontal mixing have been made by Möller [21] and Elliott and Smith [8]. If it can be assumed that horizontal mixing is proportional to the horizontal temperature gradient, with a constant coefficient of proportionality, then it can be shown that:

$$\overline{\mathbf{V}'\cdot\nabla T'}=-A\nabla^2\overline{T} \tag{2}$$

where A is called the "Austausch" coefficient (always positive according to theory), and $\nabla^2\overline{T}$ is the horizontal Laplacian of the mean temperature field (often called the thermal vorticity).

In regions of strong cyclonic thermal vorticity, as in the northeastern continental areas (see fig. 6), the eddy term is negative and, like the vertical motion term, tends to reduce the magnitude of heating given by the horizontal term alone. An opposite effect is produced in regions of anticyclonic thermal vorticity, as indicated by anticyclonic shear of the isotherms in the western oceans (fig. 6) and anticyclonic curvature in the northeastern oceans.

An attempt was made in the present study to include quantitative estimates of the eddy term as computed by equation (2), but this was abandoned in view of the obvious crudeness of this method as revealed by the studies of Möller and Elliott and Smith.

It is clear that the only way to avoid this problem completely is to compute the terms using synoptic maps and then average them over the desired time interval. This task, too laborious to accomplish on a large scale by the older "hand" methods, has recently been tackled with the aid of an electronic computer by Wiin-Nielsen and Brown (to be published). Their preliminary results for one winter month show that a region of weak cooling replaces the heating in figure 1 in eastern Canada. However, this may be due to the fact that the circulation for the chosen month (January 1959) was extremely abnormal at high latitudes.

The conclusions to be reached from the above arguments are that the strong heating centers in the northeastern continents in figure 1 should be weakened and shifted to the southeast, and that the weak heating in the southeastern oceans should be eliminated, or even replaced by centers of cooling.

4. HEAT-BALANCE METHOD

The sum of all sources of real heat gain or loss in a certain column of air of unit cross-section is equal to the flux-divergence plus the storage of heat in that column. With the aid of a continuity equation for real heat, it can be shown that the storage plus flux-divergence is equal to the total individual rate of heating. Thus, the average heating per unit mass in the column is given by the equation:

$$\overline{\frac{dq}{dt}}=\frac{g}{\Delta p}\,(\overline{Lr}+\overline{R}+\overline{P}). \tag{3}$$

Here g is the acceleration of gravity; Δp, the pressure difference between bottom and top of the column (here 500 mb.); Lr, the net heat gain due to condensation or evaporation; R, heating due to radiation; and P, that due to turbulent exchange with the earth's surface. As before, the bar overscore represents an average over a specified interval of time.

The only approximation in this equation is to omit other sources of heating or cooling which are generally considered negligible. Among these sources is the transformation between kinetic energy and heat in the frictional boundary layer.

As a matter of practical convenience, it is assumed in addition that (a) the net condensation and evaporation is measured by the heat equivalent of rain or snow reaching the ground; and (b) the radiation balance is determined by the net radiative inflow at the bottom and top of the column. In accordance with assumption (a) the quantity L in equation (3) is the latent heat of condensation or sublimation, and r is the amount of precipitation reaching the ground.

In practice the individual physical processes on the right of equation (3) are evaluated from climatological data. Thus, depending on the equations used, a certain residual effect is present, which is due to correlation in time between parameters appearing in these equations. The consequences of these "eddy" terms will be treated briefly in a later paragraph. Here it will only be stated that this is probably not very important.

In addition to being relatively free of errors associated with eddy terms, the heat-balance method has another important advantage over the thermodynamic energy equation method in that there is no tendency for compensation between individual terms, in the sense that there is a high negative pattern correlation between any two of them. In fact, just the opposite is true, as will be clarified in later paragraphs.

To compensate for these advantages, it is difficult to formulate the individual physical processes in mathematical terms. Even if this were possible, the necessary

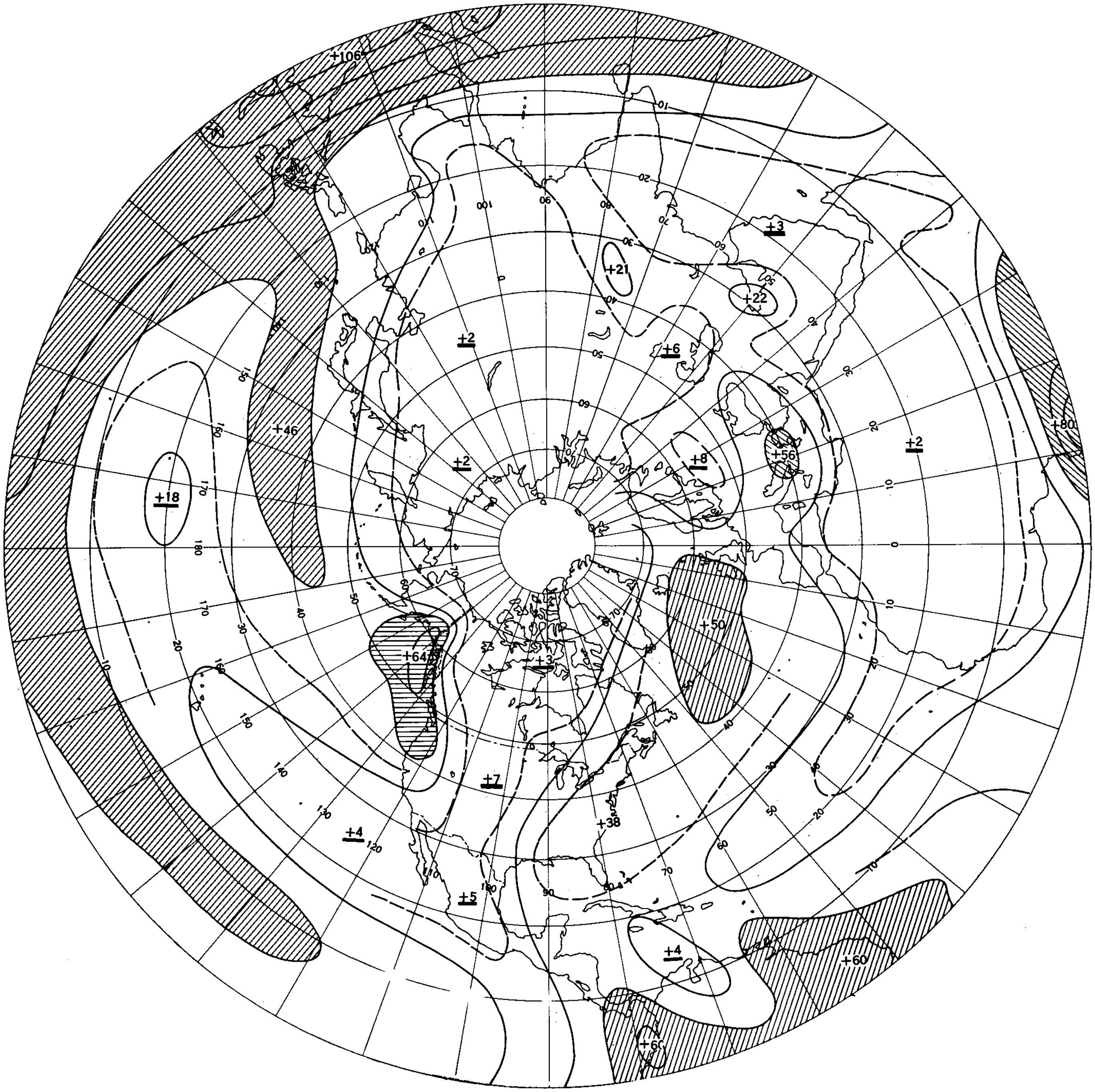

FIGURE 2.—Normal winter heating, sea level to 5.5 km. due to condensation and evaporation. See figure 1 for legend. Intermediate lines dashed, centers of relative minima underlined.

hemispheric data for evaluating them are frequently lacking.

A. EVAPORATION AND CONDENSATION

To evaluate this component of heating according to previous assumptions, it is necessary to determine the hemispheric distribution of winter precipitation, and simply convert this to heating units by multiplying it by the latent heat of condensation or sublimation. The result is then substituted for Lr in equation (3) under the assumption that all condensation takes place below the 500-mb. level.

Two separate fields were assembled, one using precipitation over the land by Landsberg [16] and over the oceans by Jacobs [14], and the second using hemispheric precipitation patterns worked up by Möller [22]. These were averaged and multiplied by a constant latent heat of condensation, equal to 600 cal. gm.$^{-1}$ The results are shown in figure 2. Strictly speaking the latent heat of sublimation should have been used in those areas where precipitation reached the ground in the form of snow. In addition, the magnitude of latent heat should be a function of the temperature of condensation or sublimation. However, these refinements would make little difference in the magnitude and pattern of heating.

Another minor discrepancy arises from the assumption that the net condensation and evaporation is measured by precipitation reaching the ground. This is a good assumption except possibly in those areas where clouds form and drift into other regions before evaporating or discharging their moisture. However, the heat content of such clouds is probably quite small, and it is difficult to isolate large areas where a systematic effect of this kind is to be found.

More important than these considerations is the question of the accuracy of the precipitation amounts. Some encouragement is gained by noting that the two precipitation fields are very similar, both in pattern and magnitude (separate charts not shown). However, this must be tempered by the realization that the authors used similar procedures and sources of data.

Over the lands, precipitation amount is determined with fair reliability from direct observation.

Over the oceans, where direct observations are lacking, precipitation amount is determined from the frequency of rainfall. This is converted to precipitation by relating it to known amounts at island and coastal stations, after taking into account the effect of latitude and geographical location. The net result is to cast doubt on the magnitude, but not the pattern, of rainfall over ocean areas.

Two other sources of error influence the computed magnitude of heating, especially in the western oceans. The first is the truncation effect, mentioned earlier, due to reading precipitation charts at widely spaced grid points. This results in a reduction in the amplitude, or difference between maxima and minima. This is partly compensated by the second error, resulting from the fact that in regions of heavy rainfall cloud tops frequently extend above the 500-mb. level. Thus, part of the condensation occurs above the 1000- to 500-mb. layer, resulting in an overestimate of heating in these regions.

If the above conclusions are accurate, namely, that at least the *pattern* of heating shown in figure 2 is correct, this helps to establish the pattern of the *total* heat budget of the troposphere. For, as pointed out by Albrecht [1], and as will be shown further on in this report, there is a high positive correlation between the precipitation pattern and that of total heating.

B. SENSIBLE HEAT EXCHANGE

The gain or loss of real (sensible) heat by direct turbulent contact with the earth's surface was obtained for ocean areas from estimates by Jacobs [14] and Budyko [5]. Jacobs made use of the so-called "Bowen formula" which is based on an assumed simple ratio between the turbulent exchange of heat and that of water vapor. The evaporation is assumed to be given by the product of the wind at observation level times the vertical vapor-pressure gradient. The sensible heat exchange is thus a function of evaporation, temperature gradient, and water vapor gradient near the sea surface.

Budyko used a formula giving the heat exchange as the product of the wind and the vertical temperature gradient multiplied by a certain exchange coefficient which is assumed to be a constant.

These formulae are regarded by many authorities as being fairly reliable, in spite of the undeveloped state of boundary-layer turbulence theory. The exchange coefficient, assumed constant, is known to depend to some extent on stability. Normal winter values of the different parameters, determined with reasonable accuracy from ships' observations, were used in evaluating the equations.

The two separate heating patterns (not shown) are in good agreement, although it must be stated that essentially the same sources of oceanic data were used.

No winter charts of sensible heat exchange over land are known to the author. However, Budyko [5] presents a chart for the year as a whole, as well as values for each month of the year for 12 stations representing certain selected climatic zones. These values are obtained from the equation for heat balance of the land surface, expressing the sensible heat exchange as the difference between radiation of all wavelengths reaching the ground and the heat of evaporation. The amount of evaporation is in turn determined as the difference between total precipitation and runoff.

In order to obtain a rough estimate of the pattern of sensible heat exchange over land areas, the present author adopted the following scheme: (a) Using Köppen's climatic classifications, the land areas were subdivided into climatic zones approximating the definitions used by Budyko. (b) Within each zone, the difference between January heating and that for the year as a whole was assumed to be the same as that for the appropriate climatic station for that zone. (c) Budyko's yearly maps were then read at standard latitude-longitude intersections, and the results of step (b) applied to these yearly means. In order to read Budyko's maps, the lines of constant sensible heat exchange were extrapolated through mountainous areas although Budyko was careful to discontinue the lines in the higher elevations.

The results of this admittedly crude procedure are plotted in figure 3 along with the average of the two ocean estimates. Only the general magnitude and sign of the computations over land have any reliability. This shows, as one might expect, that the values are in general negative over northern continents (where relatively warm air flows over a cold snow surface) and positive over the southern continents (where cold air from the north flows over warmer snow-free ground), and that the largest magnitude (−16 units) is considerably smaller than those over the oceans. The general magnitude of heat loss over the northern continents is in fair agreement with independent estimates of others. For example, Hanson [12] has re-

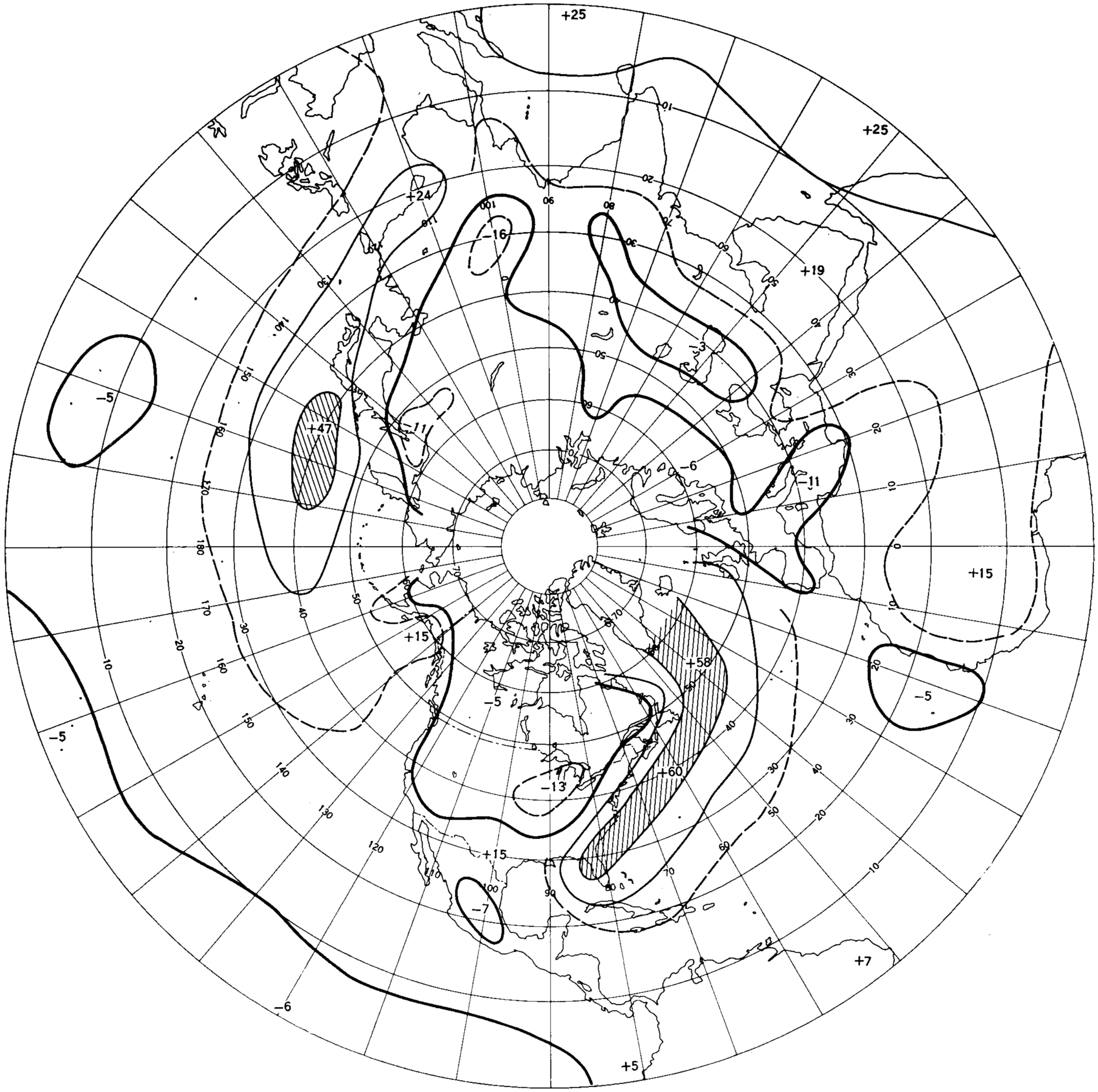

FIGURE 3.—Normal winter heating, sea level to 5.5 km. due to exchange of sensible heat with the earth's surface. See figure 1 for legend.

cently computed a value of −11 units at the South Pole in spring.

Comparison of figures 2 and 3 shows a high positive correlation between these two independent heating fields, as mentioned previously. This correlation is especially good north of 20° N.

C. THE RADIATION BALANCE

The third important component of the heat-balance is obtained by subtracting the net loss of radiation by all wavelengths at the bottom of the atmosphere from the net incoming radiation at the top. This radiation flux-divergence for the entire atmosphere is then "reduced" to the layer 1000–500 mb.

The net incoming radiation is obtained from Simpson's [28] classical computations, which appear to remain the only available source for the geographical distribution of this quantity. According to Godske et al. [11], Simpson's charts probably portray the pattern of radiation quite well, although recent advances in computation methods might well improve the magnitudes.

Simpson first computes the effective incoming short-wave solar radiation, and subtracts from this the net outgoing long-wave radiation. In his calculations, both of these quantities depend largely on cloudiness, latitude, and season. The outgoing long-wave radiation is obtained by Simpson's well-known technique of first dividing the black-body radiation curve into finite regions depending on the transparent or opaque parts of the water vapor spectrum. The outgoing radiation to space in the transparent regions depends on the normal surface temperature, while that in the opaque regions is assumed to depend on the temperature of the uppermost water vapor layer, located in the lower stratosphere. The temperature of the stratosphere is taken as a function of latitude only. When clouds are present, they are assumed to radiate as a black-body at constant temperature (260° A.). Simpson's method appears to result in too small values for the outgoing radiation, since later investigators have found that the effective upper water vapor layer lies well below the tropopause.

The net radiation loss of the air near the ground is taken from Budyko [5]*. This depends on a rather complicated chain of four empirical formulae of the type pioneered by Ångström and Brunt. The formulae depend on climatological mean values of albedo, cloud amount, the fourth power of surface air temperature, vapor pressure in the lower atmosphere, and the fraction of short-wave radiation reaching the ground with cloudy skies. The albedo depends on the latitude, season, and average type of ground cover; while the fraction of short-wave radiation reaching the ground with cloudy skies is assumed to be a function of latitude only.

In extracting data from Budyko's radiation maps the isolines were extended through mountain areas as in the case of the sensible heat. Also, after the results had been subtracted from Simpson's values and then analyzed, certain discontinuities depicted by Budyko along coastlines and the shores of inland water bodies were ignored.

This net radiative flux-divergence for the whole atmosphere was reduced to the layer 1000 to 500 mb. with the aid of London's [17] mean zonal cross-sections showing the local radiative heat loss as a function of altitude and latitude for each season. From London's cross-section for winter, a coefficient was obtained (for each latitude) equal to the ratio between the mean heating in the layer 1000–500 mb. and that for the entire layer 1000–0 mb. This was multiplied by the total flux-divergence for the appropriate latitude to obtain a reduced value. Finally, the latter was adjusted so that its zonal mean for each latitude agrees with those of London.

The final result of the various steps is shown in figure 4. Unlike the field of figure 3 the pattern correlates very poorly (although still slightly positively) with the net precipitation pattern shown in figure 2. It is easy to show qualitatively that this poor correlation is due largely to the method of reduction. This may be demonstrated with the aid of two schematic diagrams prepared by Möller [23] showing the variation with altitude of radiative flux-divergence with clear skies, and with cloudiness and rain. It is clear from these diagrams that in regions of small precipitation, where there are frequent clear skies or low clouds having tops well below the 500-mb. level, over half of the total radiative flux-divergence lies below 500 mb. Therefore in these regions (central and northern continents) the magnitude of radiative loss in the lower troposphere should be somewhat larger than that computed with a reduction factor depending on latitude alone. On the other hand, in regions of heavy precipitation, (western and northern oceans and near the equator) where cloud tops frequently reach to or above the 500-mb. level, the radiative flux-divergence shown in figure 4 must be sharply reduced because it is almost zero within and below the clouds. These modifications would clearly lead to an increase in the positive correlation between the fields of radiative heating and condensation.

No attempt has been made in this study to apply such modifications in a quantitative fashion. It was felt that this would be tantamount to "begging the question." The solution to this problem must await a fresh approach to the computation of radiative flux-divergence as a function of geography and elevation.

D. TOTAL HEATING

The sum of the three principal components of heat-balance is shown in figure 5. This is the final estimate of the normal heating of the lower troposphere by the heat-balance method. As expected, it shows a high positive pattern-correlation with the distribution of heating by precipitation (fig. 2) despite the unsatisfactory results for the radiative component (fig. 4). The lack of any major modification of the pattern is due to the fact that the radiation distribution is almost zonal, with relatively weak gradients. Correction of the field of radiation would have the effect of increasing the magnitude of the maxima and minima in figure 5 (especially the maxima) without changing the pattern greatly.

Albrecht [1] has computed the field of heating for the year as a whole, also using the heat-balance method. His pattern is quite similar to that in figure 5 over the oceans and along the eastern coasts of continents. As might be expected from seasonal changes in heating over land, his chart shows less cooling in continental interiors.

E. THE EDDY TERMS

It must be clear from the discussion in subsections A through D above that the most important question regarding the heat-balance method is that of the validity of the crude semi-empirical procedures used for evaluating the components. For this reason it seems almost beside the point to consider the possibility of additional errors due

*A similar study has been made by Bernhardt and Phillips [3]. Unfortunately, their complete radiative heat balance, including the long-wave components, was not available. However, it may be noted here that their charts of total short-wave radiation reaching the ground agree very well with those of Budyko.

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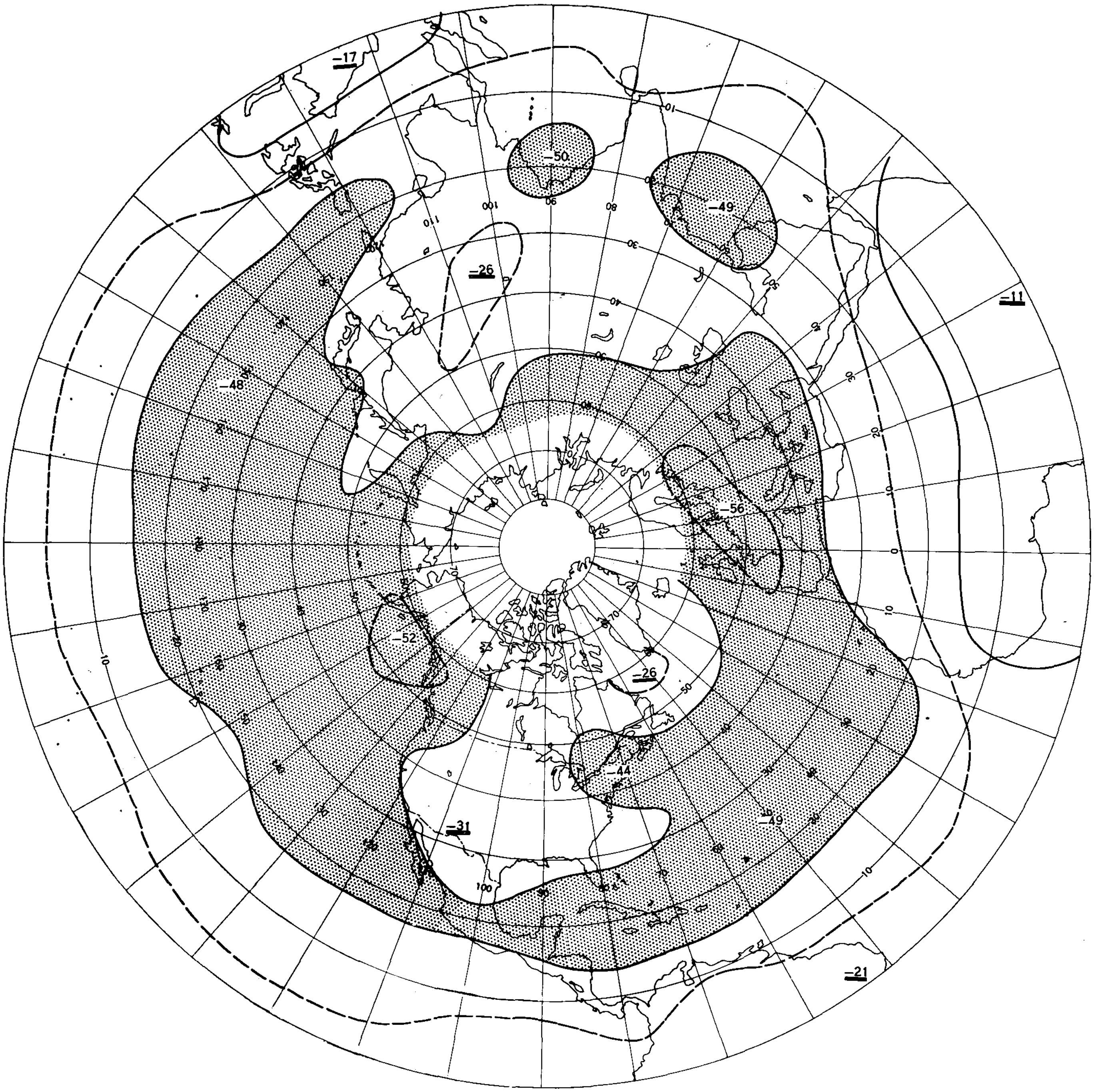

FIGURE 4.—Normal winter heating, sea level to 5.5 km. due to radiation of all wavelengths. See figure 1 for legend. Relative maxima underlined.

to correlation between the individual meteorological variables used in evaluating the equations. Furthermore the formulae for radiation and sensible heating have been designed specifically for use with climatic data, and would probably not be used if these two heating components were to be evaluated on a day-to-day basis. Nevertheless, the problem of eddy effects is so important for the thermodynamic energy equation method that it will also be raised in connection with the heat-balance method. Therefore, a brief discussion of its possible importance will be given here.

There is no eddy effect in the computation of heating by evaporation and condensation because this is obtained simply by summing the daily amounts or frequencies of precipitation. Therefore, it may be concluded that the pattern of precipitation (and presumably also the pattern of total heating) is free of errors from this effect.

The formula used by Budyko for sensible heat exchange

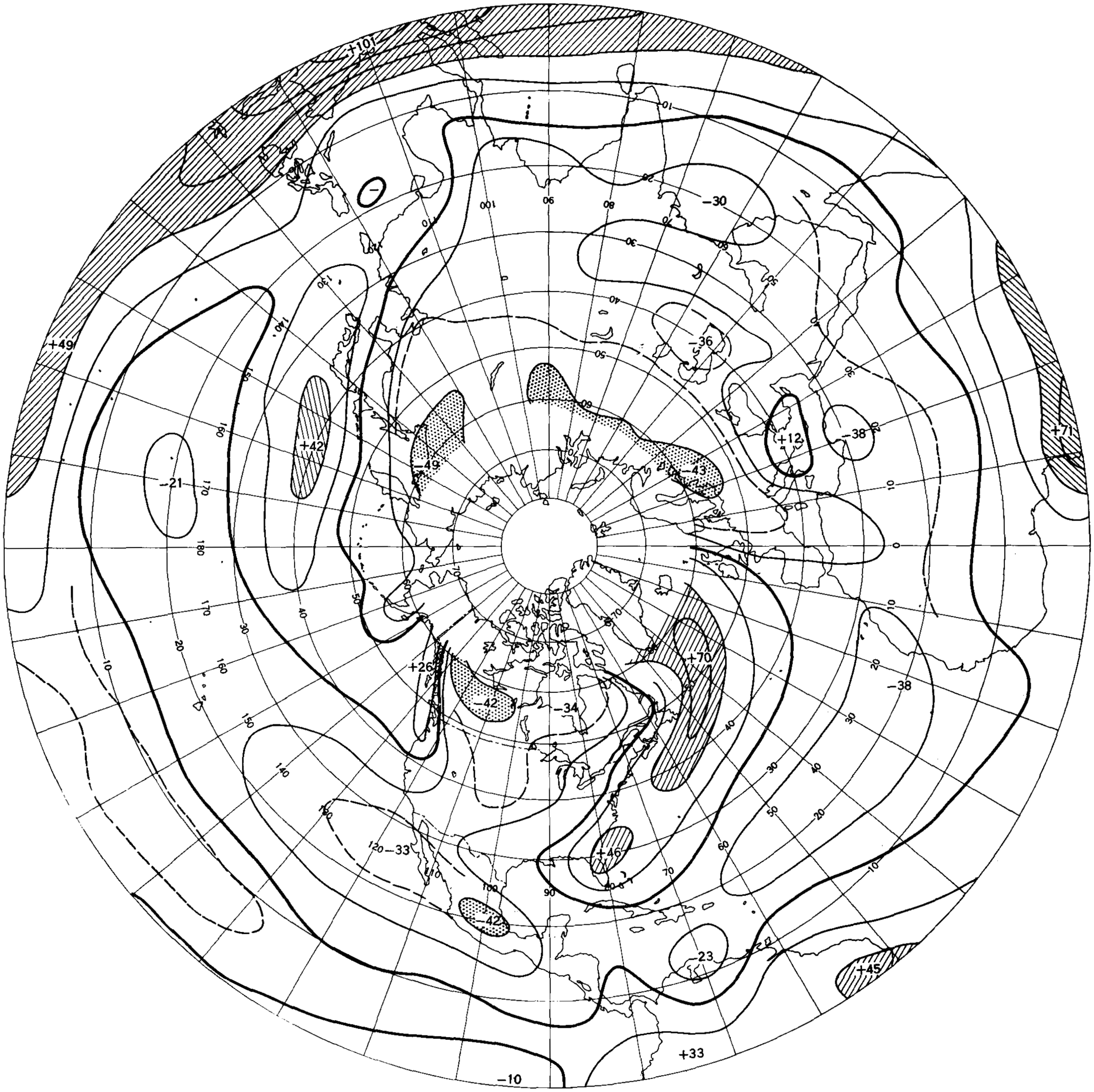

FIGURE 5.—Normal winter heating, sea level to 5.5 km., computed by the heat-balance method. See figure 1 for legend.

may be subject to some error due to a positive correlation between the exchange coefficient and the temperature gradient. This arises because the turbulence exchange coefficient becomes smaller the more stable is the lapse rate. This would lead to some increase in the estimate of heating over ocean areas portrayed in figure 3. However, Budyko [6] states that this effect is not important over oceans.

The crude estimates of this sensible heat component over land would fortunately not be further biased by eddy effects because the heating is estimated by linear heat-balance equations.

Since Jacobs used the Bowen ratio, the effect of a variable coefficient is partly eliminated. The good agreement between Jacobs' and Budyko's values over oceans provides some weak evidence that eddy effects are unimportant.

In the radiation computations there are several oppor-

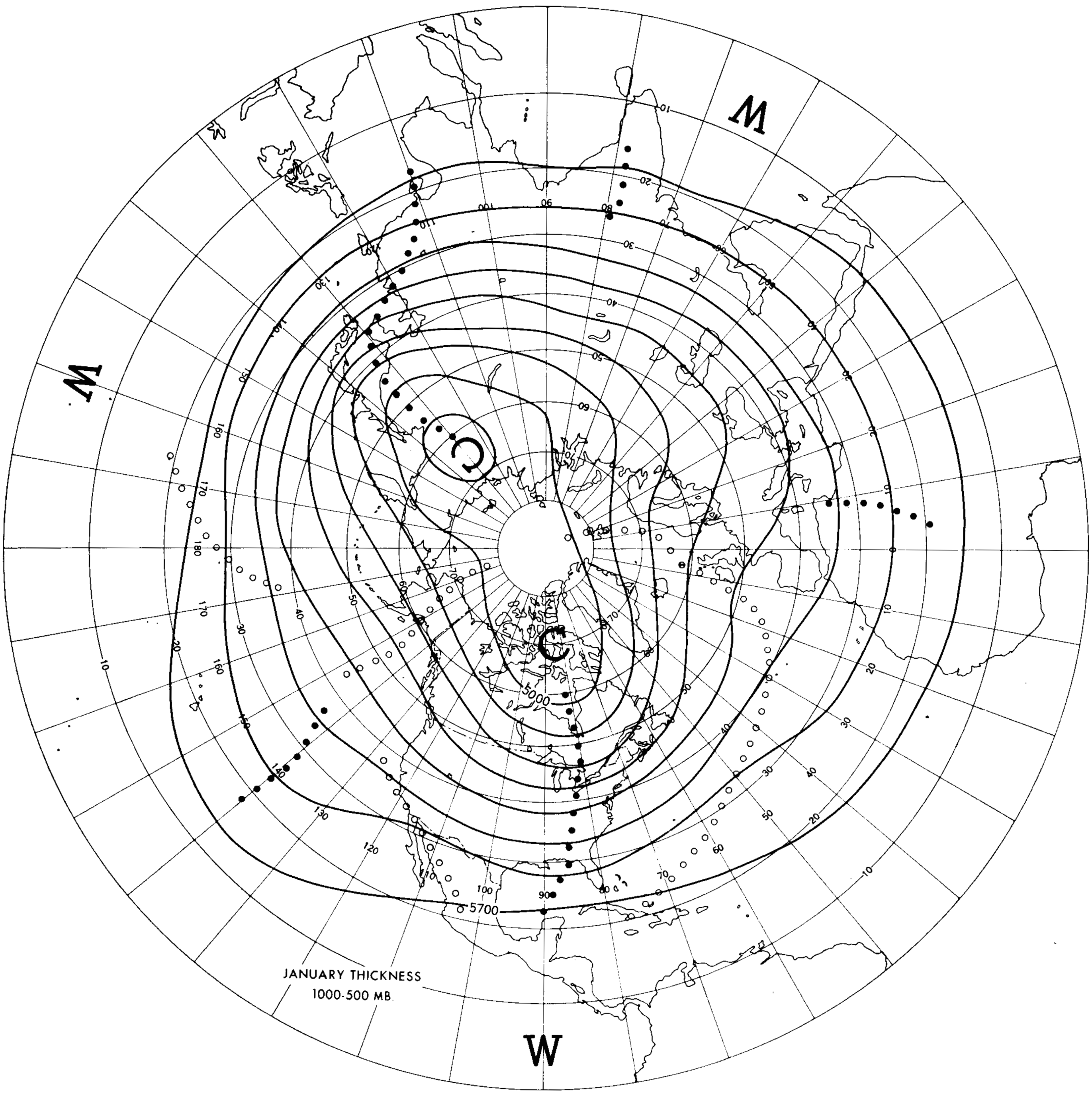

Figure 6.—Normal January thickness, 1,000 to 500 mb., after Jacobs [13]. Lines drawn for every 100 meters and labeled in meters. Centers of minimum thickness labeled C; maximum, W.

tunities for correlations to arise between parameters such as the albedo and short-wave radiation reaching the ground. Budyko [6] has attempted to make estimates of the errors arising in various steps of the radiation budget at the ground. He concludes that there is a 5 to 10 percent error in the monthly values of short-wave radiation, but that larger errors arise in the total budget because of irregular variations in albedo and surface temperature. However, in comparing estimates of total radiation with direct instrumental observations for a few Russian stations, he gets what he terms "very satisfactory" agreement. His computations also agree with the independent calculations by others.

Thus we may conclude that eddy effects play a relatively minor role in the heat-balance method.

5. THE ENERGY SOURCE FOR THE CIRCULATION

The two independent heating patterns (figs. 1 and 5) are quite dissimilar in their details, but their large-scale features are qualitatively alike, and show a similar overall

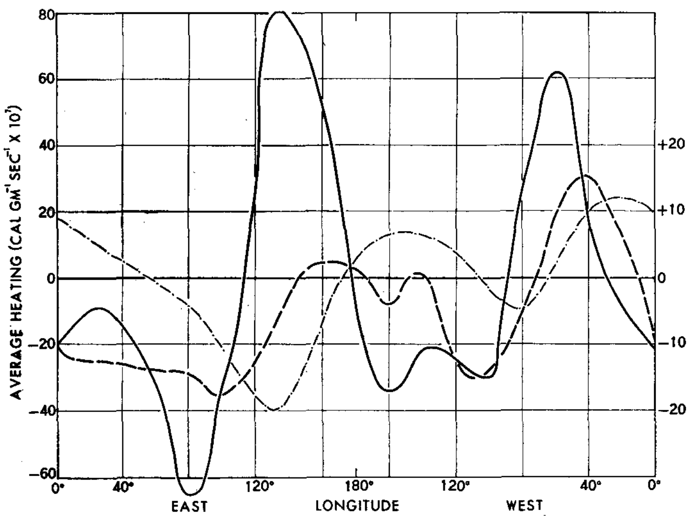

Figure 7.—Average normal winter heating and thickness in the layer 1,000 to 500 mb. and in the latitude band 30° N. to 60° N. as a function of longitude. Solid curve, heating by the thermodynamic energy equation method. Dashed curve, heating by the heat-balance method. Dash-dot curve, thickness departure from latitude average.

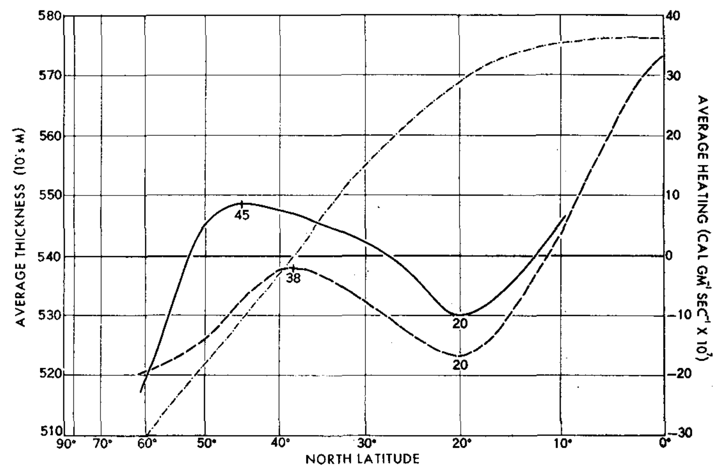

Figure 8.—Zonal averages of normal winter heating and thickness in the layer 1,000 to 500 mb. as a function of the sine of latitude. Solid curve, heating by the thermodynamic energy equation method. Dashed curve, heating by the heat-balance method. Dash-dot curve, thickness. Latitudes of maxima and minima of heating indicated.

relationship to the mean temperature field in the lower troposphere, portrayed here by the normal thickness between 1000 and 500 mb. (fig. 6). The latter has been constructed from normal data recently published by I. Jacobs [13].

Both heating patterns show a broad ring of maximum heating near the equator, associated with the highest tropospheric temperatures; maxima in middle latitudes in and to the east of the thermal troughs along the coasts of Asia and North America, and in the central Mediterranean; and cooling in and to the east of the long-wave ridges in western North America and the eastern Atlantic. Both charts show a broad region of cooling over the Eurasian continent associated with a rather flat west-to-east thermal pattern.

These large-scale relationships are brought out more clearly at middle latitudes in figure 7, which shows the average thickness and heating in the belt 30° N. to 60° N. plotted as a function of longitude. The curve of heating by the thermodynamic energy equation method agrees in almost all details with an independent computation by Wiin-Nielsen [32].

Figure 7 also emphasizes the considerable difference in magnitudes of heating and cooling by the two methods and shows an important phase difference between them. The maxima and minima of heating by the thermodynamic method are displaced 10° to 20° of longitude farther to the west. This seemingly small phase difference is quite important when considering the energy budget of the circulation. Lorenz [18] has shown that in order to get a positive transformation from thermal to potential energy, which is available for driving the circulation, there must be a positive correlation between heating and temperature. But the correlation between these two quantities, as portrayed in figure 7, is negative (−0.42) according to the thermodynamic and positive (+0.39) according to the heat-balance method. Thus, depending on which method is assumed correct, one may conclude that the semipermanent circulation in middle latitudes in winter is either energy-consuming or energy-producing. From the previous arguments in Sections 3 and 4 it is probable that the maximum heating centers of the thermodynamic method are too far west, while the pattern obtained from the heat-balance method is essentially correct. Thus, it is concluded that the semipermanent systems are energy-producing on the average. Some further interesting details of the energy budget will be given in later paragraphs.

Figure 8 shows the zonal averages of heating and thickness plotted against a sine function of latitude in order that a unit distance on the abscissa may represent a fixed area on the earth. Both of the heating curves show a positive correlation between thickness and heating south of 20° N. and north of 45° N. According to the equations of Lorenz [18], this means that in these regions there is a positive transformation between zonal heating and zonal available potential energy. The cooling at 20° N. and heating at the equator, shown in figure 8, may be identified with the sink and source of heat for the Hadley Cell, the thermally directed meridional circulation of low latitudes. The results of this study indicate that there is a positive transformation between heating and zonal potential energy at these latitudes, providing an important source of energy for the Hadley Cell. This is in agreement with a study by Tucker [30] which indicates that within the Hadley Cell there is also a positive transformation between zonal potential and zonal kinetic energy.

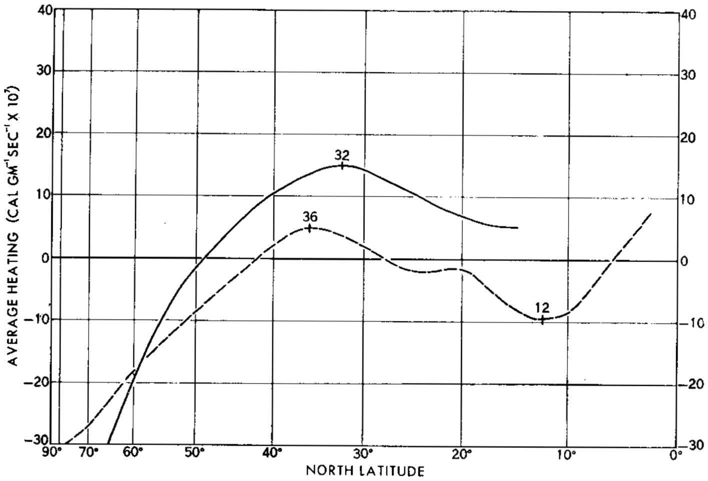

FIGURE 9.—Normal winter heating as a function of the sine of latitude after Pisharoty [27], solid curve; and after Gabites [10], dashed curve. See text for explanation.

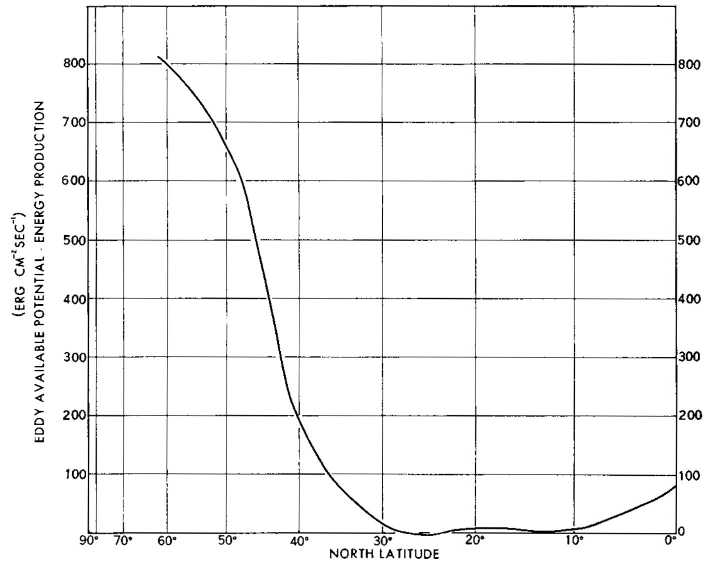

FIGURE 10.—Production of eddy available potential energy by the covariance of normal mean temperature and normal heating for winter, plotted as a function of the sine of latitude.

It is possible to provide a rough check on the validity of the latitudinal heating curves by comparing them with other studies. Two of these are summarized in figure 9, although unfortunately neither one has been computed for the same atmospheric layer, period of time, or heating components as those in figure 8. The reader must be referred to the sources for a discussion of the assumptions and approximations of these two estimates.

The first estimate, shown as the solid curve in figure 9, represents the horizontal geostrophic flux-divergence of heat in the layer 1000–200 mb. and for January and February 1949 (Pisharoty [26]). This omits the flux-divergence of heat by the meridional cells. The second estimate, shown as the dashed curve in figure 9, gives the normal heat-balance for January for the entire earth-atmosphere system (Gabites [10]). In addition to the average individual heating for the entire atmosphere, this includes heating in the oceans, measured by the latitudinal flux-divergence of heat produced by ocean currents.

Comparing these curves with those in figure 8, we note that there is good overall agreement in the location of the maxima and minima of heating. The curves of figure 9 are in better agreement with the magnitude of the heating maximum in middle latitudes obtained by the thermodynamic method, and suggest that the corresponding "maximum" for the heat-balance method is much too low. On the other hand, they are in better agreement with the location (phase) of the mid-latitude heating "maximum" for the heat-balance method. Both of these observations are in agreement with conclusions reached earlier from an analysis of the two procedures.

Finally, it is of some interest to attempt a more detailed quantitative analysis of the thermal energy budget of the semipermanent systems using the heating fields computed by the heat-balance method. The reader is referred to the original paper of Lorenz [18] for derivation of the energy transformation functions. Here, a somewhat different derivation by A. Wiin-Nielsen (unpublished lecture notes) has been used. This shows that the transformation between total heating and total available potential energy per unit area is given by the equation:

$$C=\frac{\Delta p}{S\overline{T}g}\int_S \overline{T}^* \overline{\frac{dq}{dt}}^* \,dxdy. \tag{4}$$

Here, C is a measure of the production of total available potential energy by the semipermanent systems (energy from this source which is available for maintaining the total mean circulation against friction); Δp, the pressure thickness of a thin vertical layer of the atmosphere; T and $\frac{dq}{dt}$, the absolute temperature and individual heating in the layer; and S, the area of the earth's surface for which the mean energy transformation is to be computed. The asterisk represents departure of a quantity from its area average, and the bar overscore, as before, represents an average in time.

Following Lorenz, one can divide the total energy production into two components: One is the production of eddy available potential energy (energy available for maintaining the semipermanent long waves); and the other the production of zonal available potential energy (energy available for maintaining the north-south temperature gradient). The production of eddy available energy is obtained simply by replacing $\overline{T}^*$ and $\overline{\frac{dq}{dt}}^*$ by the departure of each quantity from its zonal average; while each of these two quantities is replaced by the departure of its zonal average from its area average in order to compute the production of zonal available energy.

TABLE 1.—*Computation of total, eddy, and zonal available potential energy production per unit area in two selected latitude belts of the Northern Hemisphere for winter.* (*Units are ergs cm.$^{-2}$ sec.$^{-1}$*)

Latitude belt	Energy Production		
	Eddy	Zonal	Total
0° to 60° N	+174	+275	+449
30° N. to 60° N	+382	+223	+606

In this study it has been assumed that the mean temperature and heating do not vary in the vertical through the 1000- to 500-mb. layer, so that Δp is equal to 500 mb. Also, the mean temperature in the denominator of the coefficient has been set equal to a constant.

Figure 10 was computed from equation (4) using the data of figures 5 and 6, and shows the production of eddy available potential energy per unit area as a function of latitude. This suggests that at low latitudes the semipermanent long waves provide only a small fraction of the energy necessary for their own maintenance. Most of this energy must come from other sources, such as that generated by eddies of smaller time scale. However, north of 40° N., the energy produced by the semipermanent waves increases rapidly, until at 60° N. it is the same order of magnitude as that removed by friction. The latter is either 2000 or 4000 ergs cm.$^{-2}$ sec.$^{-1}$, depending on whether one accepts the estimates of Pisharoty [26] or of Brunt [4].

The total energy production (per unit area) and its two components have been computed for the entire area of the Northern Hemisphere between the equator and 60° N., and in the mid-latitude westerly belt between 30° N. and 60° N. The results are summarized in table 1.

Qualitatively, this table shows results which are to be expected from other considerations: in the region of the Hadley Cell and other meridional cells, a relatively large amount of zonal energy is produced. This is the bulk of the total energy production for the whole area and especially at low latitudes, while the relative importance of the zonal energy production diminishes greatly at higher latitudes. Perhaps somewhat surprising is the result that more total energy is produced per unit area at high than at low latitudes.

The relatively small magnitudes in table 1 are of some concern. The largest is only about 10 to 20 percent of the average frictional dissipation. This suggests that even at high latitudes the semipermanent circulations must be supported by energy from systems of smaller time scale, perhaps that produced in the traveling cyclones and anticyclones. However, these figures are probably much too small for two reasons: the magnitudes of the heating maxima at middle latitudes are too small, as concluded in this and previous sections; and there is a very important positive correlation between heating and temperature in the vertical, contrary to one of the assumptions made in evaluating equation (4). The vertical distribution of temperature and heating is important for the semipermanent long waves as well as for the meridional cells. This must be especially true for the Hadley Cell, where all the heat of condensation is released in the lower troposphere in the Doldrum belt, while the principal heat sink is at the top of a strong moisture discontinuity in the upper troposphere. If these two corrections could be included, they would result in a marked increase in the covariance between heating and temperature when integrated both horizontally and vertically over the selected region

6. CONCLUSIONS

There are differences in important details of the two normal heating patterns (figs. 1 and 5), which indicates that either or both of them are in error. However, they agree that the phase difference between the normal heating and temperature fields is close to 90°. Therefore, any procedure used in computing the heat budget must at least yield a good estimate of the *pattern* of heating, because a small error in pattern can change the sign of the energy transformation. Also, it must be clear that theoretical heating functions used in proposed general circulation models must also adhere to this critical phase relationship if they are to be at all realistic.

Future estimates of the thermal budget should be made by a fresh attack using the heat-balance approach, and they should include the vertical as well as the horizontal distribution of heating. Sources and sinks of energy for the summer season and for the Southern Hemisphere should also be considered.

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